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# Size and The City: Productivity, Match Quality and Wage Inequality

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## **Abstract**

This paper elucidates the impact of city growth on wage and wage inequality using a search-theoretical approach. Firms differ in capital intensity and land intensity of the jobs created. When a worker meets a job via a matching technology, a match-specific productivity level is realized and they sign a job contract when they agree with the bargaining wage. A rise in population density leads to rental increment. As a consequence, a higher expected flow profit is required for the creation of a good job. Rent-sharing ensures an increase of the average wage in the good-job sector. This, in turn, increases the reservation wage of workers in the equilibrium. Although the rental increment does not affect the setup costs in the bad-job sector, higher realized productivity level is required to cover higher reservation wage. Since only job contacts with realized productivity levels exceeding reservation productivity threshold are observed, such increase in the threshold raises also the average wage in the bad-job sector. Hence, the average productivity, the match quality and wage go up in each sector unambiguously, giving rise to urban wage premium. In addition, this paper predicts that urbanization widens residual wage inequality of a city. Existing empirical evidence is presented to support the implications of this model.

# 1 Introduction

This paper purposes to develop a simple stochastic search model to analyze the impact of city growth on labor market activity. In particular, the model explains that in a denser city, 1) the urban wage premium improves, 2) the average match quality improves and 3) the wage inequality is widened.

The model features three key elements. First, it introduces a search friction in the model economy. Therefore, the best match is not a guarantee.<sup>1</sup> Moreover, as documented in the existing empirical works (Andersson et al., 2007; Gautier and Teulings, 2009), job search and match quality matter in explaining the urban wage premium. The introduction of job search and search friction can help understand how urbanization improves wages. Second, the model is incorporated with firm heterogeneity in which a good job requires more equipment to create and is more productive than a bad job, similar to Acemoglu (2001). The proportion of each job type is endogenously determined so that the mechanism of how the composition of jobs changes in response to city growth can be analyzed.<sup>2</sup> Third, stochastic job matchings allow a well-defined wage distribution within each sector so that the model is able to elucidate the impact of city growth on wage inequality.

In our model, minimum unit of equipment is required for the establishment of a job and the equipment occupies negligible space. To install extra unit of equipment, land is needed. A good job is more capital intensive and hence occupies land while a bad job only requires a minimum amount of equipment. Firms are free to create either one of the two job types. When a worker meets a job via a matching technology, a match-specific productivity level is realized and they enter into a contract when they agree with the bargaining wage. A rise in population density leads to rental increment, and consequently, higher expected flow profit is required to create a good job. Rent-sharing ensures wages, on average, improve in the good-job sector.

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<sup>1</sup>To capture the improvement in match quality and urban wage premium in a denser city, Kim (1990, 1991) develops a model without search friction so that workers are always in the best match. Not surprisingly, when more potential match partners exist in the city, the match quality is enhanced. This leads to higher wage via wage bargaining process between firms and workers. However, the best match assumption is far from reality especially in the presence of search friction.

<sup>2</sup>In Sato (2001), technologies of jobs are horizontally differentiated and thus changes in the composition of jobs do not imply any improvement in productivity. In our model, jobs differ in productivity; therefore, urban wage premium could arise from job composition adjustment. However, such channel is blocked in Sato (2001).

This, in turn, increases the reservation wage of workers in the equilibrium. Although rental increment does not affect the setup cost in the bad-job sector, higher reservation productivity threshold is required to cover higher reservation wage of workers. Since only job contacts with realized productivity levels exceeding the reservation productivity threshold are observed in the equilibrium, such an increase in the threshold causes the average wage to rise in the bad-job sector. Hence, average productivity, match quality and wage go up in each sector unambiguously.

In the existing literature, several explanations about the urban wage premium are offered. Rauch (1993) shows that cities facilitate knowledge spillovers, enhancing human capital productivity and thus increasing wages in the human capital industry. Another theory suggests that high-skilled workers are attracted by city amenities or selectively migrate to cities with more competent workers, generating the urban wage premium in cities (Borjas et al., 1992; Yankow, 2006; Gould, 2007; Combes et al., 2008; Venables, 2011). However, Glaeser and Mare (2001) estimate the U.S. urban wage premium is about 25% even conditioning the measure of workers' human capital, reflecting such premium arises from the improvement of workers' productivity. This paper purposes to explain how this 25% match quality improvement arises from city growth conditioning labor compositions. In Andersson et al. (2007), empirical evidence supports that the improvement in match quality and thus productivity is an important source of urban wage premium. This evidence supports the implications of our model, in which urbanization, by means of rental increment, improves average productivity, match quality and thus wage in each sector.

Glaeser (1999) predicts that urban wage premium exists and wage inequality declines with density. He argues that workers can augment their human capital by interacting with those who are more talented; therefore, more vigorous productive interaction exists in the low-skilled group. He further pinpoints that such interaction rate increases with population density. As a result, average wage rises with population density, giving rise to the urban wage premium. Moreover, the model predicts that the low-skilled gain more than the high-skilled in a denser city because of higher expected gains from productive interaction, narrowing the wage inequality. This prediction, however, might not be consistent with existing empirical evidence. For example, Bacolod et al. (2009) support that productivity gain associated with agglomeration is larger for workers using stronger cognitive and people skills. Hence, urbanization would reward higher wage premium to workers in sec-

tors using cognitive skills, thereby increasing the residual wage inequality. In addition to Bacolod et al. (2009), recent empirical works (Ciccone and Hall, 1996; Korpi, 2008; Baum-Snow and Pavan, 2010) support the positive link between wage inequality and urbanization. Hence, a sound theoretical model that analyzes the mechanism of how urban wage premium and the widened wage inequality arise from urbanization is urged.

Sato (2001) introduces a job search component into his model to analyze how urban wage premium arises. In his model, higher urban density implies thicker labor market, which improves the outside option value of job-seekers when search technology exhibits increasing returns to scale. To compensate higher outside option value, higher reservation production value is required for firms and workers to match, thereby improving the match quality and thus generating an urban wage premium. Nevertheless, the urban wage premium exists in his model only if the search technology exhibits increasing returns to scale. Besides, the existing empirical works might have not yet reached a consensus. For example, empirical studies find it constant returns to scale in Petrongolo and Pissarides (2006). If it is the case, the urban wage premium does not exist in Sato's (2001) model. In contrast, our model is capable of explaining wage premium as well as greater wage inequality in a denser city by means of rental increment.<sup>3</sup> It makes economics sense that a rise in population density heightens the demand for housing and thus the bid rent. Also, Winters (2009) gives evidence that higher wages are paid in area with higher rent conditioning amenities. This evidence and the implications of our model are in accord.

Our work complements the existing models (Glaeser, 1999; Sato, 2001; Venables, 2011) in that our model predicts that urban wage premium stems from rental increment and job match. Existing empirical studies also support the implications of our model. For example, Gautier and Teulings (2009) demonstrate that two-thirds of the urban wage premium can be explained by job search. Meanwhile, as implied by Andersson et al. (2007), the urban wage premium arises mainly from the improvement in match quality. In addition to the urban wage premium, as documented in the existing literature (Ciccone and Hall, 1996; Korpi, 2008; Bacolod et al., 2009; Baum-Snow and Pavan, 2010), wage inequality grows during urbanization. This paper contributes

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<sup>3</sup>New York, London, Beijing, Tokyo and Hong Kong are examples of urbanization. Their rentals are the highest in the world. In addition, Saiz (2007) provides empirical evidence that a 1% increase in the population results in a 1% increase in the average rent and housing price in the U.S.

to the literature by elucidating the impact of city growth on residual wage inequality. This section serves as an introduction, followed by the model setup in section 2. Section 3 analyzes urban wage premium and residual wage inequality in steady state equilibrium. Section 4 concludes the paper.

## 2 Model Setup

In the model economy of population  $L > 0$ , time is continuous. All workers are infinitely-lived and risk neutral. The discount rate of workers is  $r$ . Firms are also risk neutral with discount  $r$ . All firms incur minimum cost of equipment  $k$ , which occupies negligible space.<sup>4</sup> For an additional unit of equipment, it requires firms to have an extra unit of land. Two types of vacancies exist in the economy. One type of vacancies (bad job) requires minimum cost of equipment  $k$  and negligible space (i.e.  $k_b = k$ ). Capital-intensive job (good job) requires a firm to purchase extra equipment that costs  $(\sigma - 1)k$ , where  $\sigma > 1$  (i.e.  $k_g = \sigma k$ ). Therefore, the capital-intensive job occupies  $\sigma - 1$  unit of land. Firms are free to create either one of the two types of vacancies, and purchase the sector-specific capital and land (if needed) before they create the corresponding jobs. Each worker can work for one vacancy and each vacancy can place only one worker.

Land  $T$  is fixed in the economy. In the housing market, the unit price of land is the discounted present value of rents, which is  $\int_0^\infty \Re e^{rt} dt$ . Construction cost  $C$  of a unit of land depends on the population density. The higher the population density, the higher will be the construction cost. Hence,  $C$  is increasing in the density  $L/T$ . Since the amount of land is fixed,  $C(L)$  is strictly increasing in the population density (i.e.  $C'(L) > 0, \forall L > 0$ ). The housing market is assumed to be perfectly competitive. Hence, it is straightforward to show that the rent ensures marginal revenue equals marginal cost as follows:

$$\Re/r = C(L) \tag{1}$$

Let  $u$  and  $v_j$  be the unemployment and the number of unfilled jobs  $j$  respectively. Hence,  $v = v_b + v_g$  is the total amount of vacancies in the economy. A matching technology  $M(u, v)$  is assumed to be differentiable, increasing

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<sup>4</sup>This assumption is not restrictive. Salesmen, porters, dustmen, security guards are examples of jobs that require negligible space to create a position.

in its arguments, concave, and constant returns to scale. The contact rate for a vacancy can be written as  $M(u, v)/v$ . Define  $q(\theta) \equiv M(u, v)/v$  where  $\theta \equiv v/u$  is the labor market tightness. It is straightforward to show that a labor contact rate is  $M(u, v)/u = \theta q(\theta)$ . Also, one can show that  $q(\cdot)$  is a differentiable decreasing function. Furthermore, we assume  $\lim_{\theta \rightarrow 0} q(\theta) = \infty$ ,  $\lim_{\theta \rightarrow 0} \theta q(\theta) = 0$ ,  $\lim_{\theta \rightarrow \infty} q(\theta) = 0$  and  $\lim_{\theta \rightarrow \infty} \theta q(\theta) = \infty$ .

After a worker and a vacancy  $j$  meet via a matching technology function, both parties realize the productivity level  $y_j$  of the match. They sign a contract if they agree with the bargaining wage. Otherwise the job-seeker remains unemployed and the vacancy remains unfilled in the next instant.<sup>5</sup> Once the contract is signed, this match generates a production value of  $y_j$  each instant.

The realized productivity level of a worker follows Pareto distribution  $y_j \sim \text{Pareto}(a, \underline{y}_j)$ , in which  $a > 1$  is a Pareto index, and the productivity distribution has a full support over  $[\underline{y}_j, \infty)$ .<sup>6</sup> To simplify the analysis, it is assumed that  $\underline{y}_j = Ak_j$  and  $A < r$  to capture the possibility that the realized production value can be lower than the rate of return  $rk_j$ . The cumulative distribution function is:

$$F_j(y) = \begin{cases} 1 - (\frac{y}{\underline{y}_j})^a, & \text{for } y \geq \underline{y}_j; \\ 0, & \text{for } y < \underline{y}_j. \end{cases}$$

The assumption of Pareto productivity distribution captures two properties. First, the higher the productivity, the lower will be its likelihood, i.e.  $f'(\delta) < 0$ , where  $f_j(y)$  is the corresponding density function. Second, no worker generates infinite amount of goods, i.e.  $\lim_{y \rightarrow \infty} f_j(y) = 0, \forall j \in \{b, g\}$ .

The equilibrium will be characterized through a series of Bellman equations. Let  $J_j^E(y_j)$  be the discounted present value of a worker's employment in a job  $j$ , where  $j \in \{b, g\}$ . Also, let  $J^U$  be the discounted present value of a worker being unemployed. Let  $w_j(y_j)$  be the wage of the worker in the vacancy.  $J_j^E(y_j)$  is written as:

$$rJ_j^E(y_j) = w_j(y_j) + \lambda[J^U - J_j^E(y_j)] \quad (2)$$

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<sup>5</sup>Empirical evidence strongly supports this job-contact effect. See Barron (1975) and Pissarides (1986).

<sup>6</sup>Helpman et al. (2010) make the same assumption. Readers interested in the justification of this assumption may refer to Helpman et al. (2010) for further details.

A worker with productivity level  $y_j$  receives wage  $w_j(y_j)$  and separates from the job at an exogenous rate  $\lambda$ . Similarly, we can write  $J^U$  as:

$$\begin{aligned} rJ^U &= z + \theta q(\theta) \{ \phi \int [\max\{J_b^E(x), J^U\} - J^U] dF_b(x) \\ &\quad + (1 - \phi) \int [\max\{J_g^E(x), J^U\} - J^U] dF_g(x) \} \end{aligned} \quad (3)$$

where  $\phi \equiv v_b/v$  is the fraction of unfilled bad jobs amongst all vacancies. A job-seeker receives unemployment benefit  $z^7$  and matches with a vacancy at a rate  $\theta q(\theta)$ . Also, let  $J_j^F(y_j)$  be the discounted present value of a filled vacancy that is occupied by a worker and  $J_j^V$  be the discounted present value of an unfilled job  $j$ . We can write  $J_j^F(y_j)$  as

$$rJ_j^F(y_j) = y_j - w_j(y_j) + \lambda[J_j^V - J_j^F(y_j)] \quad (4)$$

A vacancy generates revenue  $y_j$ , pays wage  $w_j(y_j)$ , and separates from a worker when an exogenous separation shock comes. Similarly,  $J_j^V$  can be written as:

$$rJ_j^V = q(\theta) \int [\max\{J_j^F(x), J_j^V\} - J_j^V] dF_j(x) \quad (5)$$

Firms are free to create jobs to exhaust all rents so that

$$J_b^V = k, \quad J_g^V = \sigma k + \int_0^\infty (\sigma - 1) \Re e^{rt} dt \quad (6)$$

Following the literature (Wasmer and Zenou, 2002; Sato, 2004; Zenou, 2009; Gautier and Zenou, 2010), wages are determined by maximizing the Nash product  $[J_j^E(y_j) - J^U]^\beta [J_j^F(y_j) - J_j^V]^{1-\beta}$  to split the matching surplus, where  $\beta \in (0, 1)$  is the bargaining power of workers. It can be found that wage  $w_j(y_j)$  is the solution of the following equation:

$$J_j^E(y_j) - J^U = \beta[J_j^E(y_j) - J^U + J_j^F(y_j) - J_j^V] \quad (7)$$

Intuitively, the matching surplus of a worker is a fraction of the total matching surplus. Using equations (2), (4), (6) and (7), the wage equations are as follows:

$$w_j(y_j) = rJ^U + \beta(y_j - rJ^U - rJ_j^V) \quad (8)$$

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<sup>7</sup>The unemployment benefit could be financed by a lump sum tax, which will not affect the result and the analysis.



Intuitively, a worker is compensated with the outside option value and a fraction  $\beta$  of the net flow profit. It can be easily verified that wage  $w_j(y_j)$  is strictly increasing in  $y_j$ . Using equations (2) and (4) as well as  $w'_j(y_j) > 0$ ,  $\partial J_j^E(y_j)/\partial y_j > 0$  and  $\partial J_j^F(y_j)/\partial y_j > 0$ , there exists a unique productivity level such that  $J_j^E(y_j) = J^U$  and  $J_j^F(y_j) = J_j^V$  respectively. Together with equation (7), it is straightforward to show that the reservation productivity thresholds of workers and jobs are identical. Denote the reservation productivity level of job  $j$  by  $y_j^R$ . Then, by definition,  $J_j^E(y_j^R) \equiv J^U$ . Since  $J_j^E(\cdot)$  is an increasing function, a worker and a job  $j$  will accept all the matches so long as the realized productivity level exceeds  $y_j^R$ . Using  $J_j^E(y_j^R) \equiv J^U$  and equation (8), the reservation productivity level is:

$$y_j^R = rJ^U + rJ_j^V \quad (9)$$

It is intuitive to notice that the reservation productivity value just covers the outside option value of a worker and a job. The reservation productivity level is higher in the good-job sector,  $y_g^R - y_b^R = (\sigma - 1)(\Re + rk) > 0$ , to cover higher capital and rental cost. A job-seeker and an unfilled vacancy  $j$  are willing to sign a contract if the realized productivity level is higher than  $y_j^R$ . Otherwise the job-seeker remains unemployed and the vacancy remains unfilled. Regarding the unemployment rate, the flow of workers back to unemployment equals the flow of job-seekers out of unemployment in the steady state. Hence, the steady state unemployment is as follows:

$$\frac{u}{L} = \frac{\lambda}{\lambda + \theta q(\theta)[\phi(1 - F_b(y_b^R)) + (1 - \phi)(1 - F_g(y_g^R))]} \quad (10)$$

Unemployment rate is strictly decreasing in  $\theta$  and increasing in  $y_j^R$ . Intuitively, as the market tightness  $\theta$  increases, workers find jobs faster. This lowers the unemployment rate. Also, an increase in the productivity thresholds  $y_j^R$  reduces the probability that jobs and workers are successfully matched, thereby increasing the unemployment rate.

### 3 Steady State Equilibrium

A steady state equilibrium is defined as value functions  $J_j^E$ ,  $J_j^F$ ,  $J^U$  and  $J_j^V$ , rent  $\Re$ , wage  $w_j$ , reservation productivity level  $y_j^R$ , a fraction of unfilled bad job  $\phi$ , unemployment rate  $u$  and labor market tightness  $\theta$  such that equations

(1)-(7), (9)-(10) are satisfied for all  $j \in \{b, g\}$ . Using equations (4), (5), (8) and (9), two zero-profit equations can be written as:

$$rJ_j^V = q(\theta) \frac{1-\beta}{r+\lambda} [1 - F(y_j^R)] [\mathbf{E}(y_j - y_j^R | y_j \geq y_j^R)] \quad (11)$$

Vacancies are created until cost equals the expected flow profit. Differentiating  $[1 - F(y_j^R)] \mathbf{E}(y_j - y_j^R | y_j \geq y_j^R) = \int_{y_j^R}^{\infty} (x - y_j^R) f(x) dx$  with respect to  $y_j^R$  using Leibniz integral rule, it is straightforward to show that  $[1 - F(y_j^R)] \mathbf{E}(y_j - y_j^R | y_j \geq y_j^R)$  is decreasing in  $y_j^R$ . Using equation (11), it is intuitive to notice that  $y_j^R$  and  $\theta$  are negatively related because higher reservation threshold reduces the match rate and thus the expected flow profit. As a result, fewer vacancies are created and thus lower  $\theta$  is resulted in the equilibrium. Regarding the existence of both types of jobs, the following holds in the equilibrium:

$$\frac{rJ_g^V}{rJ_b^V} = \frac{[1 - F(y_g^R)] [\mathbf{E}(y_g - y_g^R | y_g \geq y_g^R)]}{[1 - F(y_b^R)] [\mathbf{E}(y_b - y_b^R | y_b \geq y_b^R)]} \quad (12)$$

It can be easily shown that  $(\sigma^{a-1} - 1)\sigma/(\sigma - 1) > \Re/rk$  is a necessary condition to hold equation (12). Otherwise there exists only one type of jobs in the steady state equilibrium. I assume  $(\sigma^{a-1} - 1)\sigma/(\sigma - 1) > \Re/rk$  throughout the model to ensure  $\phi^* \in (0, 1)$ . Using Pareto distribution, equations (6), (9) and (11), the steady state reservation productivity level  $y_b^{R*}$  and  $y_g^{R*}$  are given by:

$$y_b^{R*} = \frac{(\sigma - 1)(rk + \Re)}{\left(\frac{y_g}{y_b}\right)^{\frac{a}{a-1}} \left(\frac{1}{\frac{(\sigma-1)\Re}{rk} + \sigma}\right)^{\frac{1}{a-1}} - 1}, \quad y_g^{R*} = y_b^{R*} + (\sigma - 1)(rk + \Re) \quad (13)$$

The steady state reservation productivity level  $y_j^R$  exists and is given by equation (13). Using equations (11) and (13), the equilibrium market tightness is given by:

$$rk = q(\theta) \frac{1-\beta}{r+\lambda} [1 - F(y_b^{R*})] [\mathbf{E}(y_b - y_b^{R*} | y_b \geq y_b^{R*})] \quad (14)$$

where  $y_b^{R*}$  solves equation (13). Since the right hand side of equation (14) is strictly decreasing with  $\theta$ ,  $\theta^* \in (0, \infty)$  is unique in the equilibrium. Using

equations (3), (5), (6) and (9),  $\phi^* \in (0, 1)$  can be obtained from the following equation:

$$y_b^{R*} = z + rk + q(\theta^*) \frac{\beta}{1 - \beta} \{ \phi rk + (1 - \phi)[(\sigma - 1)\Re + \sigma rk] \} \quad (15)$$

where  $y_b^{R*}$  and  $\theta^*$  are from equations (13) and (14). Since the right hand side of equation (15) is strictly decreasing with  $\phi$ ,  $\phi^* \in (0, 1)$  is unique.

**Proposition 1.** *Given any productivity level  $y \geq y_g^R$ , a worker receives a higher wage in the bad-job sector. On average, wage is higher in the good-job sector.*

*Proof.* See the appendix. □

The first implication of proposition (1) is concerned about the wage differential across sectors with identical production value. Having identical productivity level but a higher investment and rental cost, a good job will generate lower net flow profit than a bad job. Hence, a worker receives a lower wage in a good job due to rent-sharing. The second part of proposition (1) states that wage is, on average, higher in the good-job sector. Intuitively, a good job incurs a more expensive equipment cost and rental than a bad job so that it has to generate a higher expected flow profit for its creation. In equilibrium, the rent-sharing rule ensures, on average, a higher wage in the good-job sector. Since workers are homogenous, the mean wage differentials across two sectors can be interpreted as residual wage inequality. In other words, with identical observable characteristics of workers, wage is, on average, higher in the good-job sector, generating the residual wage inequality. This is in line with the evidence that more capital-intensive jobs pay higher wages (Abowd et al., 1999).

### 3.1 City Size and Labor Market

In this section, the effect of larger city size on labor market is examined. This section shows that a rise in population density leads to rental increment. City growth ameliorates match quality by means of higher rental and reservation productivity thresholds, generating the urban wage premium. In addition, this section analyzes the relationship between city size and wage inequality. Existing empirical evidence is shown to support the implications of the model.

**Proposition 2.** *A denser city has a higher reservation threshold  $y_j^{R*}$  in each sector and a lower labor market tightness  $\theta^*$ .*

*Proof.* See the appendix.  $\square$

In a denser city, the higher rental requires a good job to increase its reservation productivity threshold to cover its setup cost. Regarding the bad-job sector, rental increment does not have any impact on the setup cost of a bad job. However, the equilibrium effect of city growth increases the outside option value of a worker  $J^U$ , which will be discussed later. Since the reservation productivity level merely covers the outside option value of a worker and an unfilled job, the increase in  $J^U$  further increases both  $y_b^R$  and  $y_g^R$ . In addition, higher rental discourages the creation of good jobs, thereby reducing labor market tightness  $\theta$ . This is consistent with the empirical evidence of Detang-Dessendre and Gaigne (2009), in which they document that although the number of jobs rises with city population, the probability of a job-seeker receiving a job offer is higher in a small or medium city in France due to higher labor market tightness.

**Proposition 3.** *City growth has the following effects:*

1. *Average labor productivity  $\mathbf{E}(y_j|y_j \geq y_j^R)$  improves in each sector.*
2. *Total matching surplus, on average, improves in each sector.*  
*i.e.  $\mathbf{E}(J_j^E(y_j) - J^U + J_j^F(y_j) - J_j^V|y_j \geq y_j^R)$  goes up for all  $j \in \{b, g\}$ .*
3. *Average wage increases in each sector, generating the urban wage premium.*  
*i.e.  $\mathbf{E}(w_j(y_j)|y_j \geq y_j^R)$  increases for all  $j \in \{b, g\}$ .*
4. *Residual wage inequality rises.*  
*i.e.  $\frac{d\mathbf{E}(w_g(y_g)|y_g \geq y_g^R)}{dL} > \frac{d\mathbf{E}(w_b(y_b)|y_b \geq y_b^R)}{dL} > 0$ .*

*Proof.* See the appendix.  $\square$

Rental is higher in a denser city; hence, higher realized productivity level and expected flow profit are needed to create a good job in the equilibrium. Only job contacts with realized productivity level exceeding reservation productivity threshold are observed; hence, average labor productivity improves in the good-job sector. This causes the outside option value of workers to increase. Although rental increment has no impact on the setup cost of a bad

job, higher realized productivity level and expected flow profit are required to compensate the increase in the outside option value of workers. Similarly, such increase in the reservation productivity threshold improves average labor productivity in the bad-job sector. On average, workers receive higher bargaining wages in each sector due to rent-sharing.

This implication is in accord with the existing empirical evidence that urban wage premium improves in a denser city (Glaeser and Mare, 2001; Andersson et al., 2007; Gould, 2007; Gautier and Teulings, 2009). For example, Glaeser and Mare (2001) estimate that the U.S. urban wage premium is about 25% after conditioning experience, education and job characteristics. Controlling the measure of workers' human capital, this result clearly illustrates that the urban wage premium arises from the improvement in workers' productivity in a larger city. Gautier and Teulings (2009) give support that two-thirds of the urban wage premium can be explained by job search. Andersson et al. (2007) support that assortive matching is an important source of urban wage premium in the U.S. Hence, the implications of our model that larger city size facilitates job match by means of higher rental and thus enhances average match quality, productivity and wage are in accord with empirical evidence.

Since rental increment has a direct impact on the good-job sector in this model, the city growth effect on urban wage premium is larger in this sector. Therefore, residual wage inequality becomes wider in a denser city. Moreover, it is relatively less profitable to create a bad job in response to city growth because the expected flow profit grows faster in the good-job sector. As a consequence, the fraction of unfilled bad jobs declines. These implications and the existing empirical studies are in line with one another (Ciccone and Hall, 1996; Korpi, 2008; Bacolod et al., 2009; Baum-Snow and Pavan, 2010). For example, Ciccone and Hall (1996) give support that wage inequality is larger in a denser city while Baum-Snow and Pavan (2010) find that wage inequality and city population have a strong monotonic relationship in the U.S. In particular, their result shows that about a quarter to one-third of the overall wage inequality can be explained by city population. Their finding is consistent with the implications of this model in that wage inequality rises with city size. In addition, Bacolod et al. (2009) give support that the productivity gains associated with agglomeration are larger for workers using stronger cognitive skills, rewarding workers higher wage premium. Hence, urbanization widens residual wage inequality. In this model, a good job, on average, experiences more productivity gain from city growth than a bad job.

In addition, residual wage inequality, defined as average wage differentials of workers across two sectors, is larger in a denser city. These implications cohere with the existing empirical evidence.

## 4 Conclusion

This paper has considered the role of rent in cities. Using a search-theoretical approach, our discussion shows that urban wage premium exists in the absence of knowledge spillovers. Our model analyzes the mechanism as to how urbanization induces wage and wage inequality to grow via job match and rental increment. Although the model is simple, it gives rich economic intuition on urban wage premium. In particular, it shows that city growth improves match quality and labor productivity, thereby giving rise to the urban wage premium. Since city growth impacts disparate sectors to various extents, growth of average wage varies across sectors. This generates residual wage inequality growth in a denser city. Implications of this model cohere with recent empirical studies on urban wage premium (Glaeser and Mare, 2001; Andersson et al., 2007; Gould, 2007; Gautier and Teulings, 2009) as well as wage inequality (Ciccone and Hall, 1996; Korpi, 2008, Bacolod et al., 2009, Baum-Snow and Pavan, 2010). Works on the decomposition of wage inequality can be conducted in the future.

## 5 Proof

### 5.1 Proof of Proposition (1)

*Proof.* Using the wage equation (8), we have

$$w_g(y) - w_b(y) = -\beta(\sigma - 1)(rk + \mathfrak{R}) < 0, \forall y \geq y_g^R$$

Taking conditional expectation of the wage equation (8) and using equation (9), the difference in observable wages across two sectors is:

$$\begin{aligned} & \mathbf{E}(w_g(y_g)|y_g \geq y_g^R) - \mathbf{E}(w_b(y_b)|y_b \geq y_b^R) \\ = & \beta[\mathbf{E}(y_g - y_g^R|y_g \geq y_g^R) - \mathbf{E}(y_b - y_b^R|y_b \geq y_b^R)] \\ = & \frac{\beta(\sigma - 1)(rk + \mathfrak{R})}{a - 1} > 0 \end{aligned} \tag{16}$$

□

## 5.2 Proof of Proposition (2)

*Proof.* A rise in  $L$  causes  $\mathfrak{R}$  to rise from equation (1). It is apparent from equation (13) that  $y_j^R$  is strictly increasing in  $\mathfrak{R}$ . A rise in  $y_j^R$  reduces  $(1 - F(y_j^R))\mathbf{E}(y_j - y_j^R | y_j \geq y_j^R)$ ; hence,  $\theta^*$  has to decline to hold equation (11) in response to the increase in  $L$ .  $\square$

## 5.3 Proof of Proposition (3)

*Proof.* A rise in  $L$  increases  $y_j^R$  from proposition (2). It is easy to verify that  $\mathbf{E}(y_j | y_j \geq y_j^R)$  rises with  $y_j^R$ . Hence, average labor productivity improves in each sector.

Using equation (7),  $\mathbf{E}(J_j^E(y_j) - J^U | y_j \geq y_j^R) = \beta \mathbf{E}(J_j^F(y_j) - J_j^V | y_j \geq y_j^R) / (1 - \beta)$ ,  $\forall j \in \{b, g\}$ . Hence, the increase in average matching surplus of workers  $\mathbf{E}(J_j^E(y_j) - J^U | y_j \geq y_j^R)$  implies the increase in average matching surplus of jobs  $\mathbf{E}(J_j^F(y_j) - J_j^V | y_j \geq y_j^R)$  and the total matching surplus  $\mathbf{E}(J_j^E(y_j) - J^U + J_j^F(y_j) - J_j^V | y_j \geq y_j^R)$ .

Using equations (4) and (7),  $\mathbf{E}(J_j^E(y_j) - J^U | y_j \geq y_j^R) = \beta \mathbf{E}(y_j - y_j^R | y_j \geq y_j^R) / (r + \lambda)$ . Using Pareto distribution, it is easy to show that  $\beta \mathbf{E}(y_j - y_j^R | y_j \geq y_j^R) / (r + \lambda) = \beta y_j^R / [(a - 1)(r + \lambda)]$ , which is strictly increasing in  $y_j^R$ . Therefore, average matching surplus of job-seekers and jobs improves for all  $j \in \{b, g\}$ . The total matching surplus, on average, improves in a denser city.

Using the wage equations (8),  $\mathbf{E}(w_j(y_j) | y_j \geq y_j^R) = rJ^U + (1 - \beta)\mathbf{E}(y_j - y_j^R | y_j \geq y_j^R)$ .  $\mathbf{E}(y_j - y_j^R | y_j \geq y_j^R)$  is shown to be increasing in  $y_j^R$  and thus  $L$ . We need to prove that  $rJ^U$  increases with  $L$ . Using equations (6) and (13), it can be shown that  $drJ^U/dL = dy_b^R/dL > 0$  from equation (9). As a result, wages, on average, rise with  $L$  in each sector.

Regarding residual wage inequality, average wage differential across sectors rises with  $\mathfrak{R}$  from equation (16). Using equation (1),  $\mathfrak{R}$  rises with  $L$ .  $\square$

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